Sticky Prices in the United States

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It has often been argued that prices are sticky in the United States. However, the empirical papers that have claimed to support this view have not reflected any formal behavioral theory. This paper presents a theory that justifies price stickiness, namely, that firms, fearing to upset their customers, attribute a cost to price changes. The rational expectations equilibrium of an economy with many such firms is presented, estimated with postwar U.S. data, and tested against alternative hypotheses. The results largely support the model. Furthermore, the hypothesis that prices are not sticky is rejected by U.S. data.

I. Introduction

It is often argued that prices are sticky in the United States. In particular, various empirical papers have claimed to provide evidence that supports this view. However, these empirical papers have suffered from one major common flaw. None of them estimated equations that represented a formal theory. Therefore their results are very hard to interpret.¹ In general these studies have assumed that prices adjust at a constant rate to their desired value. The desired value was then assumed to depend on both the current and lagged values of a variety of variables (unit labor costs, the level of capacity utilization, the rental price of capital, the level of output, and the level of capital are but examples of these variables). Then these authors

¹ This problem has been pointed out already by Nordhaus (1972).

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estimated equations that made current prices a function of lagged prices and the variables purported to influence the desired prices. The fact that in the United States lagged prices help to explain current prices was taken by these authors as evidence that prices are sticky. The papers by de Menil (1974) and Eckstein and Fromm (1968), as well as the MPS model as reported by de Menil and Enzler (1972), Hymans (1972), and McCallum (1979), proceed exactly as outlined above. Eckstein and Wyss (1972), who concentrate on individual prices, also make the desired prices a function of a variety of variables and, instead of allowing lagged prices to explain current prices, they estimate their price equations by correcting for first-order serial correlation. It is hard to interpret this procedure, but it also would appear consistent with some form of price rigidity.

Of course, just the fact that lags of the variables relevant in explaining “desired” prices are significant explanators of current prices seems to point toward the existence of some sort of rigidity. This view is not shared by Sahling (1977). He explains current prices by current and lagged values of various variables. He does not include lagged prices in his list of explanators of current prices. Instead, he appears to think that prices are flexible if the current and lagged rates of return on capital do not help in predicting current prices. He views prices as relatively inflexible if neither the stocks of labor nor those of capital help explain prices. He concludes that the data support the hypothesis that prices are relatively rigid.

Domberger (1979) goes one step further. He computes the “speed of adjustment” of prices toward his measure of “desired” prices for a variety of industries. He then runs a regression of the speeds of adjustment of these industries on some indicators of industry structure. Again, even though the results seem plausible, they are hard to interpret in the absence of a theory.

Many of these authors state that prices are sticky because firms face costs of changing their prices. These costs range from the objective costs of printing new price lists to the costs borne by firms as they render their customers unhappy with recurrent price changes. If one couples the assumption that firms face costs of changing prices with assumptions about functional forms, one can estimate the parameters of the implied stochastic processes as well as test the implications of these assumptions. Moreover, this leads to parameters whose interpretation is straightforward. The purpose of this paper is to estimate and test a model in which firms face costs of changing prices.

I start in Section II by presenting the model and computing the equilibrium of an economy in which firms with market power face quadratic price adjustment costs. The implications of this equilibrium for the stochastic processes governing the joint behavior of the aggre-
gate variables (the price level, the level of output, and the money stock) are presented in Section III. Section IV estimates the model in two versions. First, it is assumed that all the prices in the United States are set by price-setting firms who face costs to changing prices. Then it is assumed that only those firms whose output belongs to the nonfarm business sector have these characteristics. The other prices are grouped in an index which is taken to be econometrically exogenous. The estimates are compared with the predictions of the model. In general, the estimates are consistent with the model. In particular, prices are shown to be sticky in the United States.

Then, the estimated equations are compared with less restricted equations to ascertain whether the U.S. data reject the model. These more general specifications do not reject the equations describing money and prices. They do, however, reject the specification of the output equation. This may be due to the neglect of the effect of relative prices on aggregate output. Section V presents some conclusions and suggestions for further research.

II. Model

The model considered here and its implications for business cycles are presented in more detail in Rotemberg (1981, in press). The economy consists of \( n \) monopolists indexed by \( i \) whose demand functions at time \( t \) are:

\[
Q_{it} = A_i \left( \frac{P_{it}}{P_t} \right)^{-b_i} \left( \frac{M_t}{P_t V_t} \right)^d, \quad i = 1, \ldots, n, \tag{1}
\]

where \( Q_{it} \) is the quantity of good \( i \) demanded at time \( t \), \( P_{it} \) is the price of good \( i \) at \( t \), \( M_t \) is the level of money balances at \( t \), and \( V_t \) is the value at \( t \) of a time-varying taste parameter. The terms \( A_i, b_i, \) and \( d \) are constants, and \( P_t \) is the price level at \( t \) which is given by:

\[
P_t = \prod_{i=1}^{n} P_{it}^{b_i/2b_1}. \tag{2}
\]

In equation (1), higher real money balances lead to a larger demand for all goods. This is a natural consequence of assuming that individuals derive utility from their holdings of real money balances.

The cost to firm \( i \) of producing \( Q_{it} \) is assumed to be given by:

\[
C(Q_{it}) = \frac{U_{it}}{2} P_t Q_{it}^2, \quad i = 1, \ldots, n, \tag{3}
\]

where \( U_{it} \) is a time-varying parameter. In particular, \( U_{it} \) could depend on the real wage at \( t \), as in Rotemberg (1981, in press). The implica-
tions for the movements of relative prices, aggregate output, and the price level are the same whether a classical labor market is included in the model or not. Therefore, for simplicity, this paper will proceed as if only goods were required to produce goods in the United States. Note that the cost functions (3) are such that the economy’s production possibilities are bounded.

In the absence of costs of changing prices, firm $i$ would charge a price $p_{it}^*$ such that the marginal revenue from sales is equal to the marginal cost of production. This price is given by:

$$p_{it}^* = s_{it} + p_t + g_i(m_t - p_t - v_t),$$

where

$$s_{it} = (a_i + \theta_i + u_{it})/(1 + b_i),$$

$$g_i = d/(1 + b_i).$$

In the expressions above, lowercase letters represent the logarithms of the respective uppercase letters and $\theta_i$ is the logarithm of the elasticity of demand over the elasticity of demand minus one and is therefore constant.

The key feature of this model is that price changes are assumed to be costly. The presence of these costs has been hypothesized by various authors, including Barro (1972), Nordhaus (1972), Sheshinski and Weiss (1977), and Mussa (1981). These costs are of two types. First, there is a fixed cost per price change which includes the physical cost of changing posted prices. Second, and in my view more important, there is a cost that captures the negative effect of price changes, particularly price increases on the reputation of firms. As stated in Stiglitz (1979), under imperfect information customers will tend to cater to firms with relatively stable price paths and avoid those firms which change their prices often and by large amounts. The reputation of firms is presumably more affected by large price changes, which are very noticeable, than by small price changes. Therefore the costs of price adjustments are assumed to be quadratic in the percentage change of prices.

The firms are now assumed to maximize the expected discounted value of the difference between revenues from sales and the sum of production costs and costs of changing prices. As shown in Rotemberg (in press), this expected discounted value can be approximated by:

$$E_t \sum_{t=1}^{\infty} \rho^{-t} [\Pi(p_{it}^*) - k_i(p_{it} - p_{it}^*)^2 - c_i(p_{it} - p_{i(t-1)})^2],$$

where $E_t$ denotes the operator that takes expectations conditional on information available at time $t$ and $\Pi(p_{it}^*)$ is the difference between the
revenues and the costs of production that would accrue to the firm if it charged $p^*_i$. The constant $k_i$ is the second term of the Taylor expansion of profits around $p^*_i$, while $\rho$ is the discount factor and $c_i$ is a parameter. At $p^*_i$, the derivative of profits with respect to prices is zero, and therefore the linear term is excluded from (5).

Maximizing expression (5) is equivalent to minimizing:

$$E_t \sum_{\tau=t}^{\infty} \rho^{\tau-t} [(p_{i\tau} - p^*_i)^2 - \frac{c_i}{k_i}(p_{i\tau} - p_{i\tau-1})^2].$$

(6)

Note that the expectations at $t$ of $p^*_i$, the price the firm would charge in the absence of costs to changing prices, are independent of the firm’s actions.

I now assume that, for all $i$, $c_i/k_i = c$. This assumption imposes one nonlinear restriction across the four firms’ specific parameters. This assumption is sufficient to ensure that all firms adjust their prices at the same speed. It is trivially satisfied when $b_i$, $c_i$, $A_i$, and $u_i$ are identical across firms.

The minimization of (6) is a recursive optimal control problem which leads to the following path for the prices that firm $i$ expects to charge:

$$p_{il+1+k} = \alpha p_{il+1+k-1} + \frac{1}{\beta \rho c} \sum_{j=0}^{\infty} \left( \frac{1}{\beta} \right)^j p^*_{il+j+k}$$

$$= \alpha p_{il+1+k-1} + \frac{1}{\beta \rho c} \sum_{j=0}^{\infty} \left( \frac{1}{\beta} \right)^j [s^i_{il+j+k} + p^i_{il+j+k}$$

$$+ g_i(m^i_{il+j+k} - p^i_{il+j+k} - v^i_{il+j+k})], \quad k = 0, 1, \ldots, \infty.$$  

(7)

The superscript $i$ denotes an expectation made by firm $i$. The quantity $p^i_{il+j}$ is the expectation held at $t$ by firm $i$ of the logarithm of the price level at $t + j$, while $p_{il+j}$ is the logarithm of the price that firm $i$ expects to charge at $t + j$. The behavioral constants $\alpha$ and $\beta$ obey the following equations:

$$\alpha < 1 < \beta,$$

(8)

$$\alpha + \beta = (1/\rho c) + (1/\rho) + 1,$$

(9)

$$\alpha \beta = 1/\rho.$$  

Therefore,

$$(1 - \alpha)(\beta - 1) = 1/\rho c.$$  

(10)

At $t$, firms actually change the leading element of the sequence given by (7), namely, $p_{il}$. In general, (7) can be interpreted as saying that,

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2 Problems of this type have been solved by numerous authors including Kennan (1979) and Sargent (1979).
because price changes are costly, the firms will move their prices slowly from their previous price to the expected $p_{t+k}^*$s.

The rational expectations equilibrium of this economy can now be computed. It requires that the firms know at $t$ the mathematical expectation of the future values of the exogenous variables $m,v,$ and $s$.

At time $t$ the firms have an expectation of the prices they will charge in the future. These are the prices the firms would actually charge in the future if they did not revise their expectations of $p, m, v,$ and $s$.

These expected future prices can be condensed to form the mathematical expectation of the future price levels. The assumption of rationality of expectations ensures that this mathematical expectation is equal to the future price levels expected by firms.

As shown in Rotemberg (1981, in press), this rational expectations equilibrium can be described by the following equation:

$$p_{t/t+k} = \gamma p_{t/t+k-1} + \frac{D}{\delta pc} \sum_{j=0}^{\infty} \left( \frac{1}{\delta} \right)^j [m_{t+j+k} - v_{t+j+k} + (S_{t+j+k}/D)],$$

(11)

where

$$D = \sum_i h_i g_i / \sum_i h_i,$$

$$S_t = \sum_i h_i \delta_{it} / \sum_i h_i,$$

$$\gamma + \delta = (D/pc) + (1/p) + 1,$$

(12)

$$\gamma \delta = 1/p,$$

(13)

$$\delta^2 - 1)(1 - \gamma) = D/pc,$$

(14)

$$\gamma < 1 < \delta,$$

and $m_{t+j+k}, v_{t+j+k},$ and $S_{t+j+k}$ are the mathematical expectations of $m_{t+k}, v_{t+k},$ and $S_{t+k}$ conditional on information available at $t$. Equation (11) describes the path of the price levels expected at $t$. The actual price levels are given by the sequence $p_{t/t}$. It turns out that $[m_t - v_t + (S_t/D)]$ is the price level that would prevail at $t$ in the absence of costs of changing prices. Therefore, (11) can be interpreted as saying that prices adjust gradually toward their expected target.

I define an index of aggregate output $Y_t$:

$$Y_t = \sum_i \frac{P_{it}Q_{it}}{P_t}. $$

(15)

3 This definition is different from the National Income account definition of GNP since intermediate goods are counted in my definition of $Y_t$. 
Recalling that in this equilibrium model output is always equal to output demanded,

\[ Y_t = \sum_i A_i \frac{P_{it}^u}{P_t} \left( \frac{P_{it}}{P_t} \right)^{-b_i} \left( \frac{M_t}{P_t V_t} \right)^a, \]

I will approximate \( \Sigma_i (P_{it}/P_t)^{1-b_i} A_i \) by a constant independent of time. That is, I approximate a weighted sum of all relative prices by a constant. This will be valid as long as relative prices do not fluctuate too widely. This approximation makes the index of aggregate output independent of relative prices. Then

\[ y_t = f + d(m_t - p_t - v_t), \tag{16} \]

where \( f = \log \Sigma_i A_i (P_{it}/P_t)^{1-b_i} \). Note that abrupt changes in the quantity of money will lead to only gradual changes in the price level and, therefore, to changes in output by (16).

III. Testable Implications for the Paths of Aggregate Variables

I will present two testable implications of the model. First, the model is consistent only with certain paths for the price level and output when money is assumed to follow a univariate autoregressive process. Second, it permits only certain paths for the price level and output when the prices of food and energy are assumed to be econometrically exogenous, while the others are set by firms whose loss function is given by (5).

The Path of Output and the Price Level When Money Is Exogenous

The model of this paper can be estimated and tested as long as assumptions are made about the paths of \( m \) and \( v \). In this section, I will assume that the \( s_t \) are fixed, while \( m \) and \( v \) follow univariate autoregressive processes of low order. The firms will be assumed to know these processes and base their pricing decisions on them.

The money stock will be assumed to follow the following process:

\[ \Psi(L)m_t = \xi_t, \tag{17} \]

where \( \Psi(L) = 1 - \Psi_1 L - \Psi_2 L^2 - \ldots - \Psi_k L^k \) and \( L \) is the lag operator. The roots of \( \Psi(z) = 0 \) are assumed to be greater than \( \rho \) in absolute value. The \( \xi_t \) are independent normal variates with mean zero and variance \( \sigma_m^2 \). In other words, it is assumed that money is exogenous and that no other economic variables “cause” \( m \) in the Granger sense. It turns out that money seems to be exogenous in this sense in the United States as demonstrated by Sims (1972), Feige and Pearce (1974), and also by me in Section IV of this paper. The taste
for money balances will be assumed to follow a first-order stochastic difference equation:

$$v_t = \lambda v_{t-1} + \nu_t,$$  \hspace{1cm} (18)

where $\lambda$ is smaller than $1/\rho$, while the $\nu_t$'s are independent normal variates with mean zero and variance $\sigma^2_t$.

Sargent (1976b) has noted that when money follows a process like (17), it is impossible to distinguish models in which output depends on the levels of $m$ and models in which output depends on the history of the unpredictable components of $m$. The model of price stickiness considered here certainly imposes restrictions on the vector stochastic process of $p$, $q$, and $m$. It is conceivable that other simple dynamic general equilibrium models impose similar restrictions. Until such other models are made explicit, however, acceptance on statistical grounds of these restrictions can be viewed as providing some support for the model of this paper.

Before these tests can be carried out, the price equation that depends on the expected levels of future money balances must be transformed so as to depend on only observables. After all, the firms make their predictions of future money balances using information on current and past levels of money balances by applying (17).

The problem is to make $\sum_{j=0}^\infty (1/\delta)^j m_{t+j}$ depend on only current and past levels of money balances. This problem has been solved in a pathbreaking paper by Hansen and Sargent (1980). They proved that

$$\sum_{j=0}^\infty \left(\frac{1}{\delta}\right)^j m_{t+j} = \left[1 - \frac{L^{-1}}{\delta} \Psi \left(\frac{1}{\delta}\right)^{-1} \Psi(L)\right]/\left[1 - (\delta L)^{-1}\right] m_t$$

$$= \Psi \left(\frac{1}{\delta}\right)^{-1} \left[1 + \sum_{j=t}^\infty \sum_{i=j+1}^k \left(\frac{1}{\delta}\right)^{i-j} \Psi_i \right] L^j m_t.$$  \hspace{1cm} (19)

Similarly, it is easy to show that

$$\sum_{j=0}^\infty \left(\frac{1}{\delta}\right)^j v_{t+j} = \sum_{j=0}^\infty \left(\frac{\lambda}{\delta}\right)^j v_t = \frac{1}{1 - (\lambda/\delta)} v_t,$$  \hspace{1cm} (20)

which converges as long as $\lambda/\delta < 1$, $\lambda < \delta = 1/\gamma$. Hence it converges if $\lambda < 1/\rho$. Therefore, the price level at $t$ is given by

$$p_t = \gamma p_{t-1} + \frac{(\delta - 1)(1 - \gamma)}{\delta} \Psi \left(\frac{1}{\delta}\right)^{-1} \left[1 + \sum_{j=1}^{k-1} \sum_{i=j+1}^k \left(\frac{1}{\delta}\right)^{i-j} \Psi_i \right] L^j m_t$$

$$+ (1 - \gamma) S + \left[\frac{(\delta - 1)(1 - \gamma)}{\delta - \lambda}\right] v_t,$$  \hspace{1cm} (21)

where (14) was used to eliminate $D/\rho c$. 

Equation (21) can be estimated along with (17), the autoregressive (AR) process for \( m \). These two equations are not independent since the \( \Psi \)'s appear in both. Equation (21) is a “structural” equation both in that it involves endogenous variables on the right-hand side and in that it has a direct behavioral interpretation. However, in contrast to the usual “structural” equations of macroeconometrics, it does not correspond to one sector’s behavior in one market. Instead, it is a property of an equilibrium which takes into account the behavior of \( n \) firms and many more consumers. In particular, it embodies assumptions about the demand functions of individuals, the cost functions of firms, and the behavior of firms.

The coefficients \( \gamma \) and \( \delta \) are identified. Unfortunately, one cannot reconstruct \( D, e, \) and \( \rho \) from knowledge of these coefficients. Instead, only \( D/e \) and \( \rho \) can be recovered by use of (12) and (13). The behavior of the aggregate variables is unaffected by a doubling of \( e \) and \( D \). As far as the price level is concerned, it is not the variable \( e \) which captures the relative pressures on charging the “right” price and on not changing prices. Instead, it is the variable \( D/e \) that represents these relative pressures. A large value for \( D/e \) means both that there is a large effect on demand when the price level is slightly wrong and that it is relatively inexpensive to change prices. Therefore, a large \( D/e \) leads to relatively fast adjustment (i.e., low values for \( \gamma \) and high values for \( \delta \)).

Note that the coefficients \( \gamma \) and \( \delta \) are independent of the monetary rule \( \Psi(L) \). Therefore, the estimates of \( \gamma \) and \( \delta \) are immune to the Lucas (1976) criticism and can be used to simulate the private sector’s response to various monetary rules. The variable \( v_t \) clearly constitutes an error term of (21) from the view of an econometrician since it is not observable. This, of course, does not make it difficult to estimate the parameter \( \lambda \) since (21) can be transformed by the usual Koyck transformation. If \( v_t \) is the only variable which makes it impossible to estimate deterministic price and output equations, then the error in the price equation ought to be proportional to the error in the output equation (16). At least the errors of both the output equation and the price equation ought to follow the same first-order autoregressive process (37).

The model of (16), (17), and (21) restricts the coefficients of the output, money forecasting, and pricing equations. To test whether these restrictions are rejected by the data, one can compare the fit of the model estimated with the restrictions to the fit of less constrained models.

As shown, for instance, in Wilson (1973), the hypothesis that \( r \) restrictions are true is acceptable for low values of the statistic, \( T(l_r - l_u) \), where \( T \) is the number of observations, \( l_u \) is the logarithm of the
determinant of the empirical covariance matrix of the errors when the estimation is carried out without the restrictions, and $l_r$ is the corresponding quantity when the estimation is carried out imposing the restrictions. In particular, under the null hypothesis that the restrictions are true, the quantity $T(l_r - l_u)$ is distributed $\chi^2(r)$, where $r$ is the number of restrictions.

The Path of the Price Level When Certain Prices Are Exogenous

In the analysis above, all the prices in the economy were assumed to be set by firms with market power who perceived price changes to be costly. However, in the United States not all prices are set by firms to whom these properties can be attributed. Instead, the model above seems best suited to explain the prices charged by the nonfarm business sector.

The dollar prices of agricultural goods are probably the prices that clear a competitive world market. Even those agricultural goods which are not traded internationally (like lettuce) are produced by competitive firms, to whom it is hard to attribute a pricing rule like (6). The prices of government services are set with consideration other than profit maximization. Meanwhile the prices of the internationally traded goods are also mostly set in world markets. In this section it will be assumed that the prices of the goods that enter the U.S. GDP, but are not produced by the business nonfarm sector, can be grouped in an index whose logarithm is $p_u$. This index will be assumed to be econometrically exogenous in the sense of Sims (1972). That is, it will be assumed that this index can best be explained by a univariate time-series process. In particular, this index will not be allowed to respond to the history of monetary innovations or even to the prices of the business nonfarm sector. This assumption of econometric exogeneity is tested below and is not rejected by U.S. data.

The price level in this case is a convex combination of $p_{ot}$ and of $\hat{p}_t$, the average price charged by the nonfarm business sector:

$$p_t = \omega p_{ot} + (1 - \omega) \hat{p}_t,$$

(22)

$$\hat{p}_t = \frac{\left( \sum_{i=1}^{n} h_i p_{u \cdot} \right)}{\left( \sum_{i=1}^{n} h_i \right)}$$

and $\omega = h_{u \cdot} / \sum_{i=1}^{n} h_i$. Using the technique which led to (11), we can write the dynamics of $\hat{p}$ as:
\[ \dot{p}_t = \gamma \dot{p}_{t-1} + \frac{D}{\rho c} \sum_{j=0}^{\infty} \left( \frac{1}{\delta} \right) j \left[ m_t + j - v_t + j - \frac{S}{D} + \frac{\omega(1-D)}{D} \dot{p}_{t+j} \right], \]  

(24)

where \( \gamma < 1 < \delta \),

\[ \delta + \gamma = 1 + \frac{1}{\rho} + \frac{D}{\rho c} + \frac{\omega(1-D)}{\rho c}, \]  

(25)

and

\[ \delta \gamma = \frac{1}{\rho}. \]  

(26)

Therefore,

\[ (\delta - 1)(1 - \gamma) = \frac{D}{\rho c} + \frac{\omega(1-D)}{\rho c}. \]  

(27)

Hence the private nonfarm business price index does not just depend on the future money stocks but also on the future exogenous component of the price level. The set of exogenous prices is a determinant of the long-run equilibrium private business price index for two reasons. First, the composite good whose price is exogenous is an input into production, and an increase in its price will, in the long run, be partially passed on to the consumers. Second, a change in the exogenous price affects the level of real money balances and, hence, aggregate demand.

In the normal case in which \( D \) is smaller than one, the adjustment of \( \dot{p} \) is faster the larger is the component of the price level which is exogenous. When a large fraction of the prices are exogenous, changes in money balances or in the exogenous prices exert large pressures on the endogenous prices leading them to adjust faster.

Let the exogenous prices be described by

\[ \Lambda(L) \dot{p}_{o_t} = \zeta_t, \]

\[ \Lambda(L) = 1 - \Lambda_1 L - \Lambda_2 L^2 - \ldots - \Lambda_b L^b, \]  

(28)

where the roots of \( \Lambda(z) = 0 \) are larger than \( \rho \) in absolute value and the \( \zeta \)'s are independent normal variates with zero mean and variance \( \sigma^2_\zeta \).

Then, using the result of Hansen and Sargent (1980) that was referred to above and letting \( Z = D/\delta \rho c \), one obtains:
\[
\hat{\pi}_t = \gamma \hat{p}_{t-1} + \Psi \left( \frac{1}{\delta} \right)^{-1} Z \left[ 1 + \sum_{j=1}^{k-1} \sum_{i=j+1}^{k} \left( \frac{1}{\delta} \right)^{i-j} \Psi_i \right] L^j m_t + (1 - \gamma) S \\
+ \frac{(\delta - 1)(1 - \gamma)}{\delta} - Z \left( \sup\Gamma \left( \frac{1}{\delta} \right)^{-1} \right) \left[ 1 + \sum_{j=1}^{b-1} \sum_{i=j+1}^{b} \left( \frac{1}{\delta} \right)^{i-j} \lambda_i \right] L^i p_{0t} (29)
\]

\[
+ \frac{Z\delta}{\delta - \lambda} \tau_t.
\]

Equation (29) makes the private nonfarm business price index a function of observables only. In particular, lagged values of \( m \) and \( p_0 \) help explain \( \hat{p} \) only because they help explain the future values of \( m \) and \( p_0 \).

Four coefficients, namely, \( \omega, \rho, D, \) and \( c \), enter the private sector’s maximization problem. Unfortunately, we can only recover three \( (\gamma, \delta, \text{and } Z) \) nonlinear functions of these coefficients. In particular, \( \omega \) is not identified, and the response of the economy to a change in \( \omega \) cannot be simulated without additional information.

IV. Estimation and Testing of the Model Using Aggregate Data

A. Generalities

To estimate the aggregate model of Sections II and III, it is first necessary to decide which observable variables best represent the theoretical constructs \( m, p, \) and \( q \). To focus on the cyclical fluctuations of output the presence of growth was neglected in the theoretical development. Therefore, the estimation is carried out with detrended variables whose mean has also been removed. The money stock that enters into the demand function (1) is assumed to be M1. The index of aggregate output \( Q \) is assumed to behave like GNP even though \( Q \) is supposed to include all the outputs of intermediate goods sections. The price level is represented by the GDP deflator.\(^4\) The variable \( p_0 \), the index of the prices that are assumed to be exogenous, is a weighted average of the prices of the sectors other than the private nonfarm business sector. The weights are given by the proportion of these sectors in total GDP. For the purpose of estimation, all variables are in natural logarithms as predicted by the theory.\(^5\)

Before the model is estimated, one question must be addressed. What equation ought to be used both by the rational agents and by

\(^4\) I have also estimated the model using the consumer price index both unadjusted and seasonally adjusted. The latter led to results essentially identical to those obtained using the GDP deflator. Instead, the use of seasonally unadjusted data produced much poorer results, which suggests that the seasonality of prices is not due exclusively to the seasonality of money.

\(^5\) A data appendix including the derivation of the series actually used is available from the author upon request.
econometricians when forecasting $m$ and $p_0$, the variables which are not under the control of price-setting firms? Here it is postulated that the best forecasting equations are given by parsimonious univariate autoregressions like (17) and (28). However, if the endogenous variables helped to predict $m$ and $p_0$, rational firms would employ them for this purpose. Therefore, the following subsection is devoted to checking that, indeed, the endogenous variables do not significantly help in predicting $m$ and $p_0$. Then it proceeds to present the parsimonious autoregressions which are taken to be the forecasting equations used by rational agents in the United States.

In the subsequent sections the model is estimated in two versions. First, in subsection C, all prices are assumed to be set by firms with loss function (5). Second, in subsection D, only those firms that produce nonfarm output are assumed to be exogenous. The estimates of these sections are the result of estimation via maximum likelihood. The DFP and MINOPT algorithms were used to obtain convergence.

**B. The Prediction of $m$ and $p_0$**

This subsection begins by presenting evidence suggesting that the univariate representations (17) for $m$ and (28) for $p_0$ are indeed appropriate descriptions of these series. These equations are, therefore, likely to be the equations used by firms when forecasting $m$ and $p_0$.

The firms in this economy are trying to predict the future values of $m$ and $p_0$ using all available current information. Therefore, they would use as predictors any lagged values of the other endogenous variables if these values helped predict $m$ and $p_0$. By the definition of Granger causality, the endogenous variables are not useful predictors of $m$ and $p_0$ if and only if they do not cause $m$ and $p_0$. I test the null hypothesis that $m$ and $p_0$ are exogenous using the technique of Sargent (1976a), which was already proposed in the original paper by Granger (1969). It consists of regressing $m$ and $p_0$ on their own past and on lagged values of other variables which potentially cause $m$ and $p_0$. $F$-tests can then be computed to determine whether these other variables are significant predictors of $m$ and $p_0$. These regressions and tests are presented in table 1. The null hypothesis of lack of causality can be accepted at the 5 percent level if the reported $F$-statistic is below 2.29. Hence I can accept that $m$ and $p_0$ are not caused by the other variables in the model.\(^6\)

\(^6\)The fact that money is not caused by but causes income has been discussed by numerous authors (Sims 1972; Feige and Pearce 1974; Sargent 1976a). It must be noted that in a later paper Sims (1980) observed that interest rates, which are absent from this analysis, do appear to cause money.
<table>
<thead>
<tr>
<th>Time Period</th>
<th>Y</th>
<th>X</th>
<th>α₁</th>
<th>α₂</th>
<th>α₃</th>
<th>β₁</th>
<th>β₂</th>
<th>β₃</th>
<th>β₄</th>
<th>β₅</th>
<th>D-W</th>
<th>R²</th>
<th>F-Statistic (All βₖ = 0)</th>
<th>Sum of Squared Residuals</th>
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</thead>
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<tr>
<td>1948:II–1979:II</td>
<td>m</td>
<td>1.53</td>
<td>-.64</td>
<td>.08</td>
<td>-.10</td>
<td>.03</td>
<td>-.003</td>
<td>.03</td>
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<td>.97</td>
<td>.731</td>
<td>2.895 E-3</td>
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<td></td>
<td>(.10)</td>
<td>(.18)</td>
<td>(.11)</td>
<td>(.05)</td>
<td>(.08)</td>
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<td>(.08)</td>
<td>(.05)</td>
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<td></td>
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</tr>
<tr>
<td>1948:II–1979:II</td>
<td>m p</td>
<td>1.59</td>
<td>-.69</td>
<td>.07</td>
<td>-.06</td>
<td>-.02</td>
<td>.26</td>
<td>-.23</td>
<td>.05</td>
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<td>.97</td>
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<td>(.18)</td>
<td>(.10)</td>
<td>(.09)</td>
<td>(.16)</td>
<td>(.17)</td>
<td>(.16)</td>
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<tr>
<td>1948:II–1979:II</td>
<td>m p₀</td>
<td>1.61</td>
<td>-.78</td>
<td>.15</td>
<td>.01</td>
<td>.01</td>
<td>-.05</td>
<td>.08</td>
<td>-.05</td>
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<td>(.03)</td>
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<td>m p₀</td>
<td>1.59</td>
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<tr>
<td>1948:II–1979:II</td>
<td>m</td>
<td>1.58</td>
<td>-.70</td>
<td>.09</td>
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<td>. .</td>
<td>. .</td>
<td>. .</td>
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<td>. .</td>
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<td>(.10)</td>
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<td>. .</td>
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<td>. .</td>
<td>. .</td>
<td>. .</td>
<td></td>
</tr>
<tr>
<td>1948:II–1979:II</td>
<td>p₀ y</td>
<td>1.01</td>
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<td>-.11</td>
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<td>-.24</td>
<td>-.23</td>
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<td>.74</td>
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<td>4.127 E-2</td>
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<td>(.14)</td>
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<td>(.19)</td>
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<td>(.29)</td>
<td>(.17)</td>
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<tr>
<td>1948:II–1979:II</td>
<td>p₀ m</td>
<td>.93</td>
<td>-.07</td>
<td>-.13</td>
<td>.07</td>
<td>.37</td>
<td>-.38</td>
<td>.33</td>
<td>-.28</td>
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<tr>
<td>1948:II–1979:II</td>
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<td>.43</td>
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<td>-.35</td>
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<td>4.068 E-2</td>
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<td>(.50)</td>
<td>(.29)</td>
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<td></td>
</tr>
<tr>
<td>1948:II–1979:II</td>
<td>p₀</td>
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<td>-.08</td>
<td>-.14</td>
<td>. .</td>
<td>. .</td>
<td>. .</td>
<td>. .</td>
<td>. .</td>
<td>2.03</td>
<td>.73</td>
<td>. .</td>
<td>4.306 E-2</td>
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<td>. .</td>
<td>. .</td>
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<td></td>
</tr>
</tbody>
</table>

**Note.**—These regressions were performed on quarterly data. Standard errors are in parentheses.
This leads me to assume that the best forecasting equations for \( m \) and \( p_o \) are the parsimonious representations of (17) and (28). These are obtained using the methods suggested by Box and Jenkins (1970).

The series \( m \) can be parsimoniously represented by an AR(2) process of the first difference of \( m \). In other words, (17) becomes

\[
(1 - \Psi_1 L - \Psi_2 L^2)(m_t - m_{t-1}) = \xi_t. \tag{30}
\]

The OLS estimates of equation (30) are given by:

\[
m_t - m_{t-1} = 0.62(m_{t-1} - m_{t-2}) - 0.13(m_{t-2} - m_{t-3}) + \zeta_t, \\
(0.09) \quad (0.10)
\]

D-W = 1.92, \( R^2 = 0.308 \), period = 1948:1–1979:II,

where the standard errors are in parentheses.

Finally, the process followed by \( p_o \) can also be represented parsimoniously as an AR(2). Equation (28), therefore, becomes

\[
(1 - \Lambda_1 L - \Lambda_2 L^2)p_{ot} = \zeta_t. \tag{31}
\]

The OLS estimates of (31) are given by:

\[
p_{ot} = 1.057p_{ot-1} - .267p_{ot-2}, \\
(0.087) \quad (0.087)
\]

D-W = 2.05, \( R^2 = 0.710 \), period = 1948:1–1979:II.

C. All Prices Are Endogenous

In this subsection I will estimate the system composed of the money forecasting equation (30), the price-level equation (21), the output equation (16), and the equation that gives the evolution of the taste for money balances over time (18).

Since a weighted average of relative prices which could be serially correlated and time varying is assumed to be constant in order to derive the output equation (16), this equation might well be misspecified. Therefore, the estimation of (16) and (21) jointly could lead the misspecification of the output equation to pollute the estimates of the parameters of the price equation. This would be true whether one imposed the restrictions that the estimated \( \lambda \)'s be the same in both equations or not. The model was thus estimated in three versions. First, it was estimated without the output equation (specifications I and II). Then it was fitted including the output equation and imposing the cross-equation restriction on the estimate of \( \lambda \) (specification

Note that the \( \bar{\Psi} \)'s are not equivalent to the \( \Psi \)'s of (17) since the former are coefficients that apply to the first difference of \( m \). Instead, the relationship between the \( \bar{\Psi} \)'s and the \( \Psi \)'s is given by \( \bar{\Psi}_1 = 1 + \Psi_1, \bar{\Psi}_2 = \Psi_2 - \Psi_1, \) and \( \bar{\Psi}_3 = -\Psi_2. \)
III). Finally, (16) and (21) were estimated jointly, but $\lambda_1$, the estimate of $\lambda$ from the price equation, was allowed to be different from $\lambda_2$, the estimate of $\lambda$ from the output equation (specification IV). This last specification might be appropriate if the technology parameter $S_t$ were not fixed. In this case the residuals of the output and price equations are not proportional to each other. Accepting the hypothesis that $\lambda_1$ is equal to $\lambda_2$ supports the view that the only shocks to the system are taste shocks. Instead, accepting the hypothesis which is embedded in specification I that $\lambda$ is zero suggests that most of the "cyclic" properties of $\gamma$ can be attributed to changes in the money stock.

The model also imposes three inequality restrictions on the parameters $\gamma$, $\delta$, and $d$, which for computational convenience were not imposed in the estimation. The roots $\gamma$ and $\delta$ of the equation that characterizes prices must have the property that $\gamma < 1$ while $\delta > 1$. Furthermore, the coefficient $d$, which measures the elasticity of aggregate output with respect to money balances, must be positive. These inequalities can, of course, be tested one by one using the estimates of the standard errors of the coefficients.

The estimates of the coefficients as well as various test statistics are presented in table 2.

The Walrasian equilibrium of this model would be observed if $\gamma$ were equal to zero and $\delta$ equal to infinity. This would, of course, require that the only explanator of the price level be the contemporaneous level of money balances. Such a hypothesis is rejected by all the estimating equations, thereby lending credibility to the notion that prices are sticky in the United States.\textsuperscript{8} Furthermore, the adjustment of prices is shown to be very slow. This is evidenced by the high values of $\gamma$, the coefficient of lagged prices in the price equation.

This slowness is due to one of two causes: Either the effect of real money balances on demand is small enough to cause the penalties that accrue from charging the wrong prices to be minimal, or the subjective costs of charging prices are high. This can be seen by computing the underlying behavioral parameters $\rho$ and $D/c$ from (13) and (14).

Using specification II, the preferred specification, the estimated value of $D/c$ (0.08) is indeed very low as required for prices to move slowly.\textsuperscript{9} This specification is preferred for two reasons: First, it is

\textsuperscript{8} In a sense this test could overstate the importance of sticky prices since it forces the Walrasian equilibrium to be a static model. Instead in the presence of costs of adjusting the capital stock of the sort discussed by Lucas and Prescott (1971), the competitive equilibrium will have dynamic features. However, insofar as the model of this paper is not rejected by less constrained specifications, it is reasonable to test the implications of forcing all prices to adjust instantaneously.

\textsuperscript{9} The implied estimates of $\rho$ are somewhat small (on the order of 0.002). This low value for $\rho$ is the reflection of the rather large coefficient estimates of $\delta$. These ensure
### Table 2

**Estimation of the Model to Explain the Path of the GDP Deflator**  
*(Quarterly Data 1948:1–1979:II)*

<table>
<thead>
<tr>
<th>Specification</th>
<th>Price and Money Equations</th>
<th>Price, Money, and Output Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>Coefficients:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Psi_1$</td>
<td>0.619</td>
<td>0.623</td>
</tr>
<tr>
<td></td>
<td>(0.092)</td>
<td>(0.092)</td>
</tr>
<tr>
<td>$\Psi_2$</td>
<td>-0.132</td>
<td>-0.137</td>
</tr>
<tr>
<td></td>
<td>(0.097)</td>
<td>(0.097)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.946</td>
<td>0.920</td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td>(0.032)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>780.884</td>
<td>2140.600</td>
</tr>
<tr>
<td></td>
<td>(0.126)</td>
<td>(60.945)</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>0.556</td>
<td>0.903</td>
</tr>
<tr>
<td></td>
<td>(0.078)</td>
<td></td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>...</td>
<td>0.888</td>
</tr>
<tr>
<td>$d$</td>
<td>...</td>
<td>0.806</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.199)</td>
</tr>
</tbody>
</table>

**Single-equation statistics:**

- Eq. (30) $R^2$: 0.308, 0.308, 0.290, 0.288
- Eq. (30) D-W: 1.920, 1.925, 1.818, 1.823
- Eq. (21) $R^2$: 0.974, 0.983, 0.943, 0.939
- Eq. (21) D-W: 0.903, 2.194, 1.794, 1.799
- Eq. (16) $R^2$: ... , ... , 0.892, 0.888
- Eq. (16) D-W: ... , ... , 1.258, 1.248

**Global statistics:**

- $T(l_u - l)$: 5.494*, 1.600*, 3.931*, 3.364*
- $T(l_1 - l)$: ... , ... , 23.197, 22.630
  - [6] [5]
- $T(l_2 - l)$: 62.572, 13.117*, 68.645, 68.072
  - [12] [11] [30] [29]

---

Note: Standard errors are in parentheses. The numbers in brackets are the numbers of restrictions tested and are thus equal to the number of degrees of freedom of the relevant $\chi^2$ distribution. An asterisk means that the value of the statistic is lower than the value to the left of which lies 99 percent of the relevant $\chi^2$ distribution.

Superior to specification I since $\lambda$ is significantly different from zero, which thus suggests that money is not the only determinant of the fluctuations of real money balances and output. Second, the output equation does appear to be misspecified, and therefore its separate estimation is warranted. The first sign of this misspecification is that expected future money balances are not important determinants of current prices. Instead, only current money balances appear to be important in this respect. In turn, this suggests that firms are not overly concerned by the future and that they have a high discount rate.
the D-W statistic corresponding to the output equation is quite low. Furthermore, the hypothesis that the estimate of \( \lambda \) from the price equation \( \lambda_1 \) is the same as the corresponding estimate for the output equation \( \lambda_2 \) is rejected as can be seen by comparing the \( l \)-statistics corresponding to specifications III and IV.

That the potential misspecification of the output equation affects the joint estimates of the system can be seen by noting that the estimates for \( \delta \) are below unity when the output equation is included. This runs counter to the model’s prediction that the “unstable” root \( \delta \) is larger than unity. Note that when the output equation is excluded, the estimates for \( \delta \) are always larger than one.

I now proceed to test these regressions against alternative hypotheses which impose less structure on the data. These less restricted specifications could arise either if the model of firm behavior was true and the expectations were not computed rationally or if the expectations were computed rationally but firms chose their prices by pursuing different objectives.

The models estimated in table 2 have many more explanatory variables than coefficients. The pricing equation in the first column has four explanatory variables (lagged prices, current, and two lags of money), while the money forecasting equation has two explanatory variables. However, only four coefficients are estimated (\( \gamma, \delta, \overline{\Psi}_1, \) and \( \overline{\Psi}_2 \)). Therefore, the model imposes two nonlinear restrictions on the coefficients of six explanatory variables. It is natural to test these cross-equation restrictions by comparing each specification to a specification that uses the same explanatory variables but does not restrict the coefficients. The logarithm of the determinant of the covariance matrix of the errors of these unrestricted regressions is \( l_0 \). The tests are carried out by computing the quantities \( T(l_0 - l) \) and contrasting them with the \( \chi^2 \) distribution with \( r \) degrees of freedom. Here \( r \) is the difference in the number of coefficients used in the computations of \( l_0 \) and the number of coefficients used in the computation of \( l \). As can be seen in table 2, the unrestricted estimates do not reject the model.

Next, the fit of those specifications which include the output equation is compared with the fit of an even less restricted model which allows output to be explained by two lags of output as well as by the current and lagged real money balances. This was done because the output equation may well be misspecified by the exclusion of a serially correlated variable. The logarithm of the determinant of the covariance of the errors from estimating this more unrestricted model is \( l_1 \). Indeed, the hypothesis that each explanatory variable has its own coefficient and that output is explained by two lags and current and lagged real money balances rejects the model when the output equa-
tion is estimated jointly with the other equations. This represents further evidence that the output equation is misspecified.

Finally, I contrast the system composed of (16), (18), (21), and (30) with an extremely unconstrained model, namely, a fourth-order vector autoregression. The purpose of this comparison is to allow the model to be rejected by models which, while theoretically eclectic, appear to fit the data very well. The vector autoregression is estimated using ordinary least squares as suggested by Sims (1980). The test of the model as a set of restrictions on the vector autoregression is given by the statistic $T(l_2 - l)$. The versions of the model in which the output equation is excluded are not rejected by the vector autoregressions. Instead, as would be expected after comparing $l_t$ with $l$, when the output equation is included, the vector autoregression with four lags rejects the model with 99 percent confidence.

This section leads to two main conclusions. First, prices do appear to be sticky in the United States. Second, the restrictions on the paths of the price level and money balances implied by the model are not rejected by U.S. data.

D. Some Prices Are Exogenous

In this subsection, \( \tilde{p}_t \), the deflator for the product of the business nonfarm sector, is assumed to be the result of prices charged by monopolies which face costs of price adjustment. The rest of the prices are assumed to be exogenous and follow (31). The model as it will be estimated thus includes equations (16), (18), (29), (30), and (31).

Before proceeding with this estimation, however, one must determine what the appropriate variable for \( p_t \) is in the output equation (16). The GDP deflator cannot be written as in (22) for any fixed \( \omega \) since the proportion of GDP accounted for by the business nonfarm sector has been increasing steadily in the postwar period. If the “correct” deflator of money balances is the GDP deflator, the estimation of (16) is straightforward while the pricing equation (29), which is derived from (22), is subject to errors of measurement. If instead the “correct” deflator satisfies (22) for some \( \omega \), the pricing equation is correctly specified and the relevant output equation is:

\[
y_t = d[m_t - \omega p_{ot} - (1 - \omega)\tilde{p}_t - v_t].
\]

This equation identifies the weight \( \omega \).

Six versions of the model were therefore fitted to U.S. data. Since the output equation may be misspecified, it was fitted separately in specifications I and II to avoid the potential pollution of the estimated parameters of the other equations. Specification I explores the effects
of forcing $\lambda$ to be zero, while specification II relaxes this constraint. Specifications III and IV use the GDP deflator as the price index which households take into account when computing their real money balances. Instead, specifications V and VI force that price index to be a convex combination of $\pi$ and $\tilde{\pi}$, as in (22) and (32). While specifications III and V force the estimate of $\lambda$ to be the same across the price and output equations, specifications IV and VI relax this requirement on the grounds that the technology parameter may be time varying.

Table 3 presents both the estimates corresponding to the various versions of the model and the statistics that can be used to judge their acceptability. For all the versions, the estimates satisfy those inequality restrictions which the model implies but which were not imposed during the estimation. In particular, $\gamma$ is positive and smaller than one, $\delta$ is greater than one, and $Z$ and $\tilde{d}$ are positive. The hypothesis that there are no costs to changing prices is rejected. Moreover, prices are very “sticky” and adjust slowly as evidenced by the high values of $\gamma$. These are caused either by a small effect of real money balances on demand (low $D$) or by high costs of changing prices, $c$. Indeed, the point estimates of $D/c$ are around 0.2.\footnote{The estimates for the discount factor are, once again, somewhat small (around 0.3).}

Once again the parameter $\lambda$ is significantly different from zero. Furthermore, the hypothesis that the parameter $\lambda$ is the same in both the price and output equations is not rejected in either of the specifications of the output equation. These facts lend credence to the notion that changes in tastes are important determinants of the cyclic movements of output.

The output equations continue to show symptoms of misspecification. Both of its versions have low D-W statistics. All versions of the model which include the output equations are rejected by models which do not constrain the coefficients and have a second lag of output as an explanator of output. They are also rejected by fourth-order vector autoregressions, as the high values of $T(l_2 - l)$ for specifications III through VI indicate.

Another unfortunate feature of the output equation is that when (32) is fitted jointly with the other equations, it leads to an estimate for $\omega$ which has the wrong sign and a large standard error. However, this estimate is not significantly different from the rather reasonable one obtained by fitting the output equation separately, as can be seen from the following regression:

$$y_t = .476[m_t - 0.128\tilde{p}_t - (1 - 0.128)p_t],$$  
$$\begin{array}{ccc} 
(0.096) & (0.091) & (0.091)
\end{array}$$

$$\begin{array}{cc}
\lambda = .902, & R^2 = .903, \ D-W = 1.23, \ \text{period} = 1948:1-1979:II. \\
(0.036)
\end{array}$$
While the output equation appears to be misspecified, the opposite is true for the equation describing the price level. Specification II, which includes the output equation while allowing \( v_t \) to follow a first-order autoregression, not only is not rejected by a model which includes the same explanatory variables without constraining their coefficients but is also accepted at the 99.2 percent significance level against a vector autoregression.

The results for this case are thus mixed. On the favorable side, the coefficients satisfy the model's restrictions, and the specification of the pricing equation is basically not rejected by less restrictive hypotheses. Furthermore, the hypothesis that prices are fully flexible is rejected. On the other hand, the output equation appears to be misspecified, and the construction of a price index that reflects the postulates of the model has probably not been achieved.

V. Conclusions

This paper presents evidence related to a specific model of sticky prices. In this model, prices are sticky because firms face a subjective cost to changing their prices. They are sticky in the sense that they are closer to those prices which prevailed the period before than they would be if no agent's decisions at \( t \) depended on prices at \( t - 1 \). The model incorporates a number of specific assumptions about the functional forms of the demand functions, production functions, etc. It therefore significantly reduces the set of outcomes that do not contradict it.

I estimate the equilibrium paths of the price level and output implied by the model. These equilibrium paths must satisfy two types of testable restrictions. First, the estimated parameters must satisfy certain inequalities. Second, the model predicts that if one were to estimate more parameters by adding explanatory variables, these added variables would not significantly contribute to the explanation of the price level and output. The estimation of this aggregate model was carried out both under the assumption that all prices are set by firms that face a subjective cost to changing prices and under the assumption that some prices are exogenous. The latter assumption produced results that are slightly more consistent with the model. In general, the estimated parameters satisfy the restrictions imposed by the model. Furthermore, when the output equation is excluded, less restricted models do not reject the model of this paper. Finally, the hypothesis that prices adjust instantaneously, under the maintained hypothesis about the functional forms of the demand and production functions, is rejected by the data. These three facts are taken to be
TABLE 3

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<th>Coefficients:</th>
<th>( \overline{\Psi}_1 )</th>
<th>( \overline{\Psi}_2 )</th>
<th>( \Lambda_1 )</th>
<th>( \Lambda_2 )</th>
<th>( \gamma )</th>
<th>( \delta )</th>
<th>( D/pc )</th>
<th>( \lambda_1 )</th>
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<td>( \overline{\Psi}_2 )</td>
<td>( \Lambda_1 )</td>
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Note.—Standard errors are in parentheses. Numbers in brackets are the relevant degrees of freedom for the null hypothesis that the estimates satisfy the constraints imposed by the model.

* The value of the statistics is lower than the value to the left of which lies 99 percent of the relevant $\chi^2$ distribution.

** The critical value for the $\chi^2(28)$ is 48.2. Therefore, the null hypothesis is almost not rejected with 99 percent confidence.
evidence that the model ought to be taken seriously as an explanator of aggregate phenomena in the United States.

On the other hand, the output equation appears to be misspecified, since when it is included the model is rejected by some of the models that are less restricted. This might be due to the neglect of the costs which are associated with changing output.

Since this model does not just make predictions about the stochastic processes followed by the aggregate variables, but instead starts by making predictions about individual prices and quantities, it would be desirable to estimate and test this model using microeconomic data. In particular, the ideal procedure would be to estimate jointly the equilibrium (represented by the movement of the aggregate variables) and the objective functions of individual firms (represented by the movement of firm-specific variables).

Less ambitious undertakings would include the estimation of the effect on the aggregate variables of both particular exogenous prices and shifts in the labor supply curve. The former could be achieved by disaggregating the consumer price index, the latter by estimating jointly a labor supply curve.

References


——. “The Observational Equivalence of Natural and Unnatural Rate Theories of Macroeconomics.” *J.P.E.* 84, no. 3 (June 1976): 631–40. (b)


